

# Models

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# Models

- abstract from details
- concentrate on functionality, properties, ... that are considered important for a specific system/application
- use model to analyse, prove, predict, ... system properties
  
- models in engineering disciplines very common, not so in CS
- we'll see many models in lecture: “Real-Time Systems (winter term)”
- today: models to analyse fault tolerance techniques
- objective: understand the need for careful understanding of models

# Fault Tolerance

- Techniques how to build reliable systems from less reliable components
- Fault(Error, Failure, ....): synonymously used for “something goes wrong”  
(more precise definitions and types of faults in SE)

# Properties

- **Reliability:**
  - $R(t)$ : probability for a system to survive time  $t$
- **Availability:**
  - $A$ : fraction of time a system works

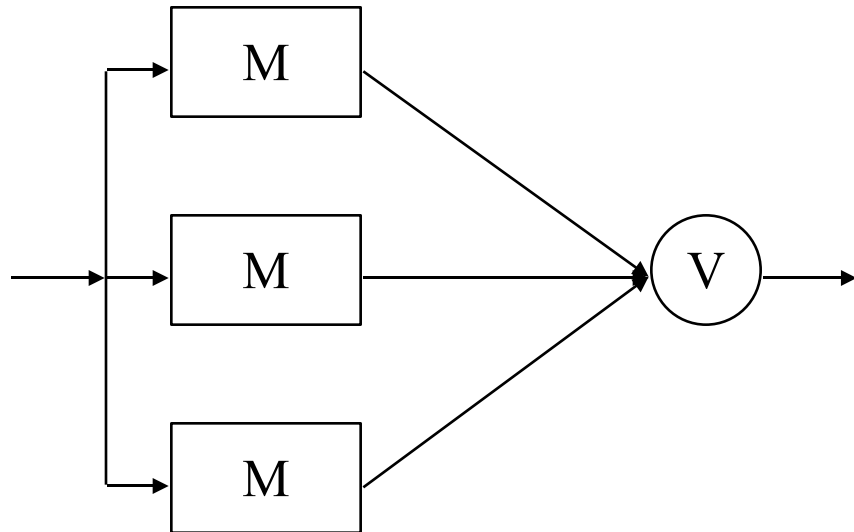
# Fault Tolerance: key ingredients

- Fault detection and confinement
- Recovery
- Repair
  
- Redundancy
  - information
  - time
  - structural
  - functional

# Examples: RAID, Triple Modular Redundancy

John v. Neumann

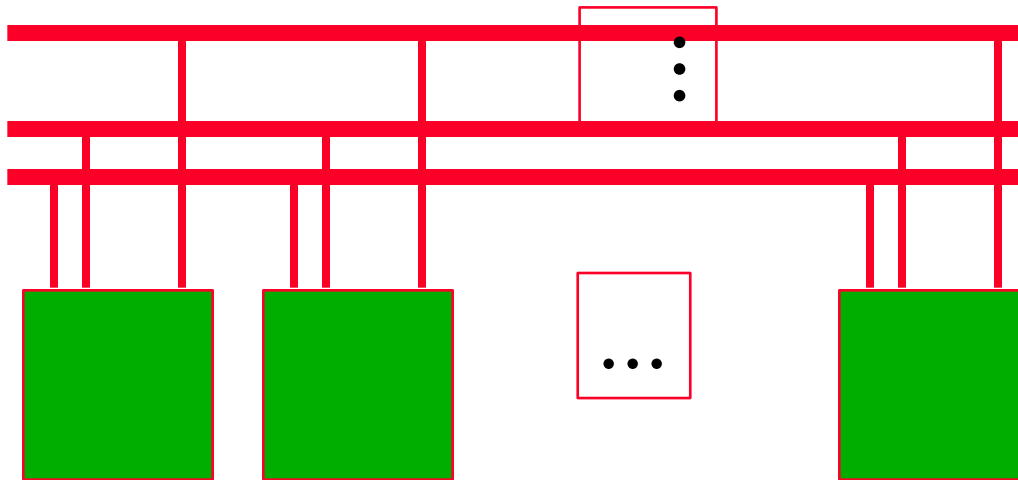
*Voter: single point of failure*



Can we do better ->  
distributed solutions ?

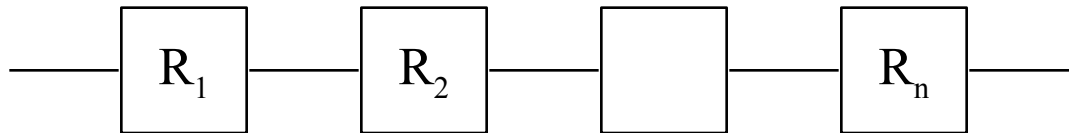
# Limits(mathematical) of Reliability, Variant 1

Parallel-Serial-Systems  
(Pfitzmann/Härtig 1982)



# Reliability Models

Serial System:



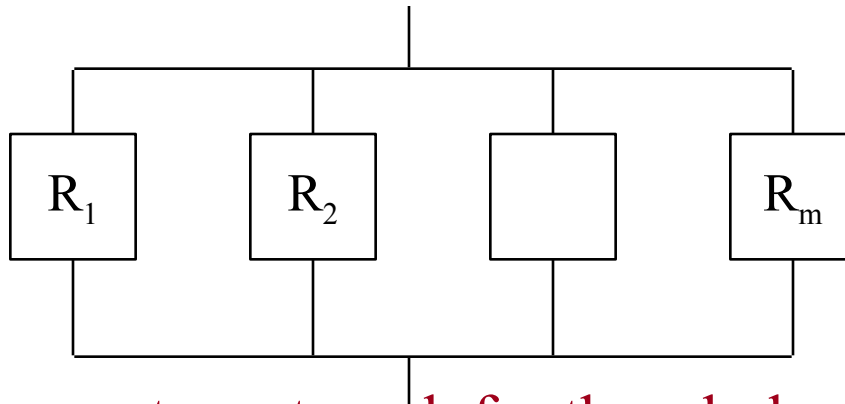
each component must work for the whole system to work

$$R_{whole} = \prod_{j=1}^n R_j$$



# Reliability Models

## Parallel System

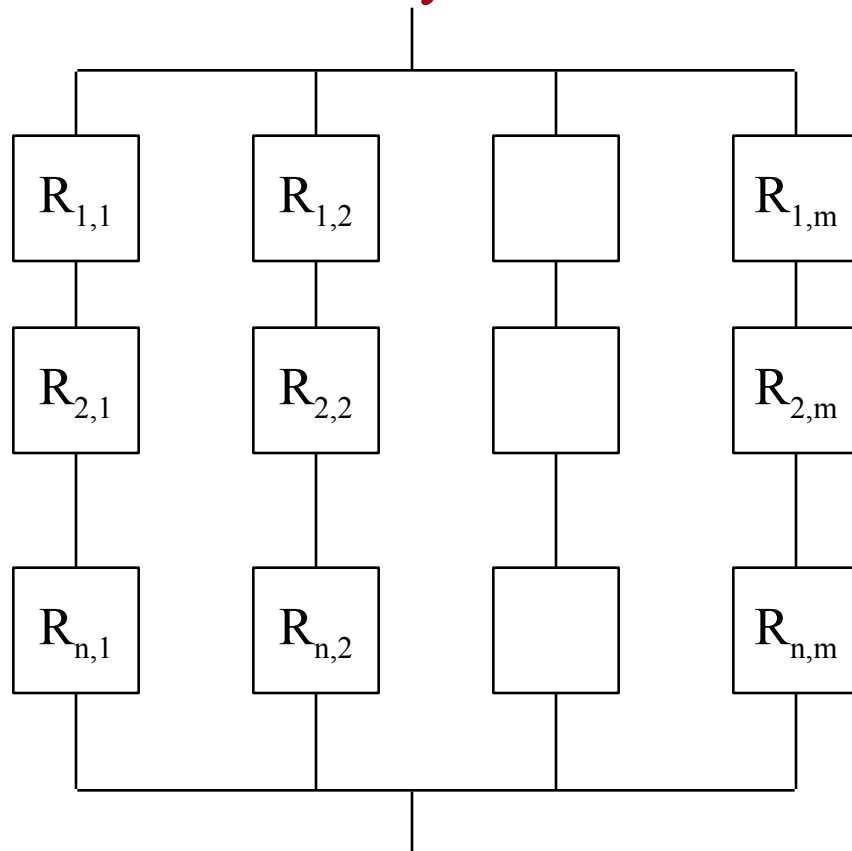


one component must work for the whole system to work  
each component must fail for the whole system to fail

$$R_{whole} = 1 - \prod_{i=1}^m (1 - R_i)$$

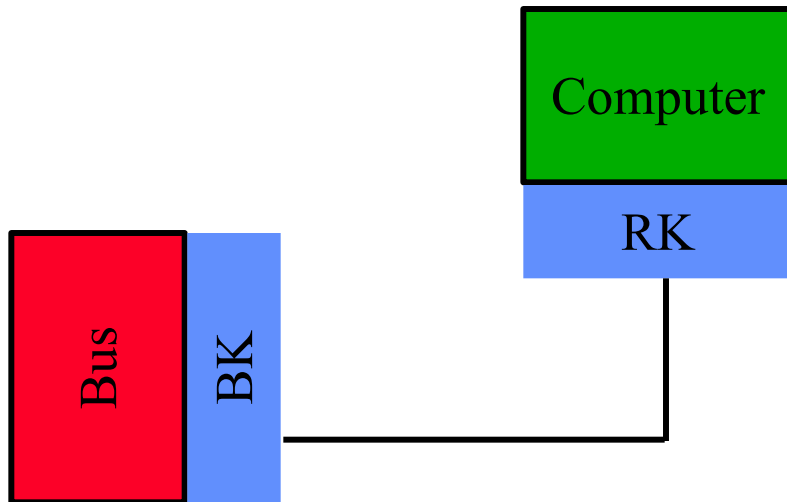
# Reliability Models

## Serial-Parallel System



$$R_{whole} = 1 - \prod_{j=1}^m \left( 1 - \prod_{i=1}^n R_{i,j} \right)$$

# Our Example



## Fault Model::

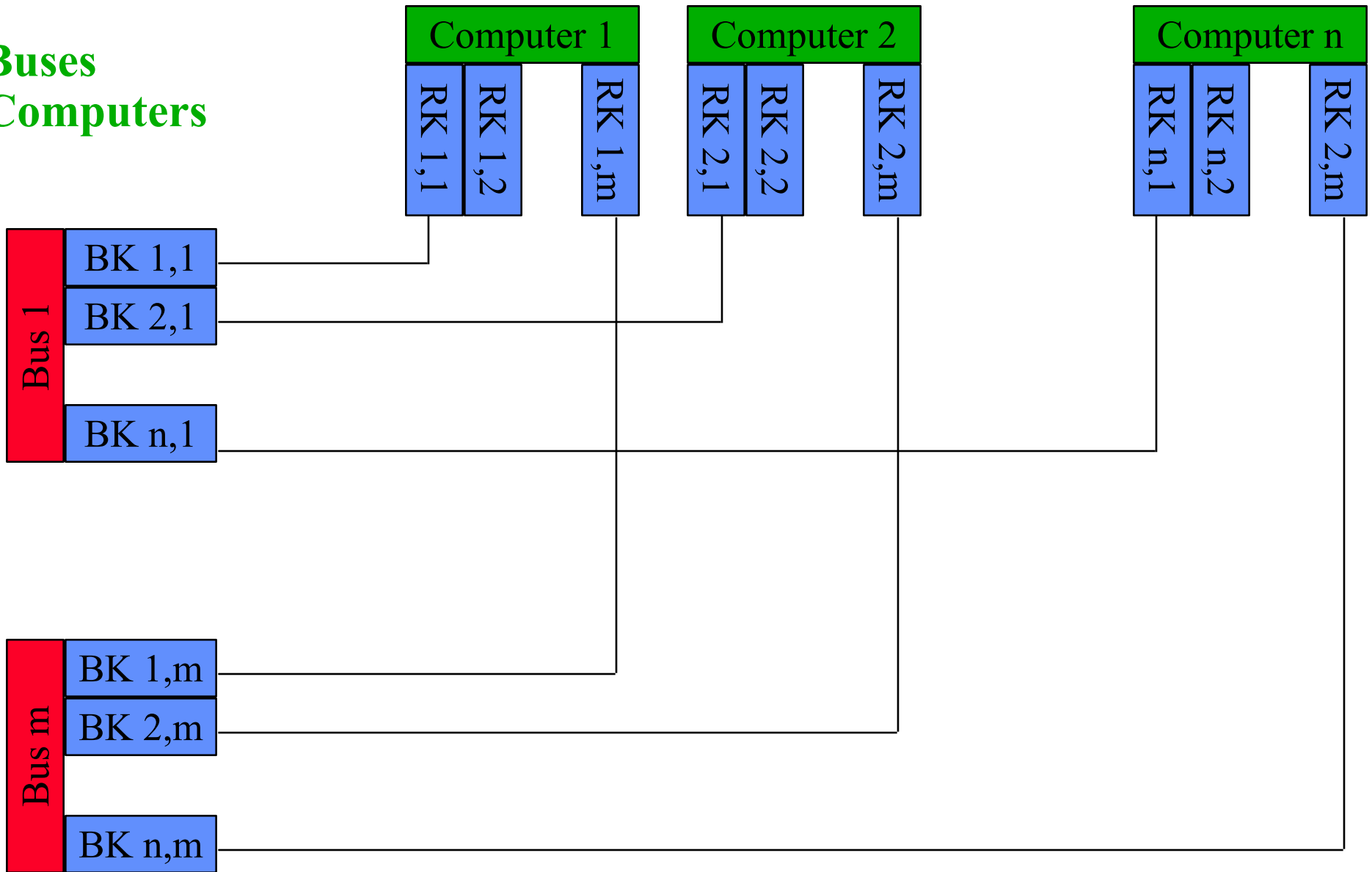
„Computer-Bus-Connector“

can fail such that Computer and/or Bus also fail

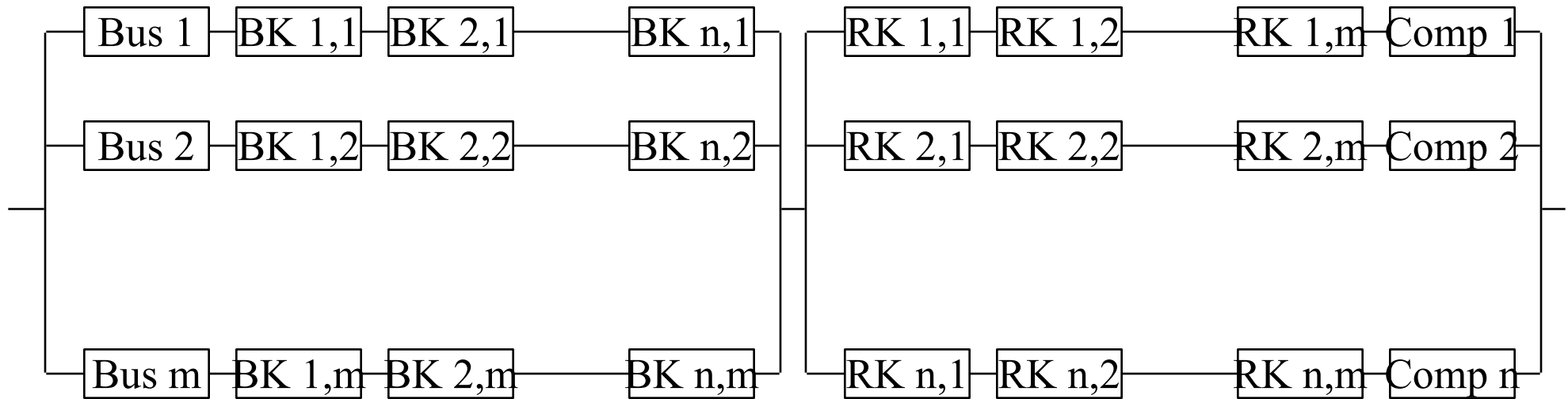
therefore we model:  
conceptual separation of connector into

- RK: Computer-Connector, whose fault also breaks the Computer
- BK: Bus-Connector, ...

**M Buses**  
**N Computers**



## Model for m,n



$$R_{whole}(n, m) = \left(1 - \left(1 - R_{Bus} \cdot R_{BK}^n\right)^m\right) \cdot \left(1 - \left(1 - R_{Computer} \cdot R_{RK}^m\right)^n\right)$$

then:  $R_{RK}, R_{BK} < 1$ :  $\lim_{n, m \rightarrow \infty} R(n, m) = ??$

# Limits(mathematical) of Reliability, Variant 2

System built of Synapses (John von Neumann, 1956)

Computation and Fault Model:

Synapses deliver „0“ or „1”

Synapses deliver with  $R > 0,5$ :

- with probability  $R$  correct result
- with  $(1-R)$  wrong result

Then we can build systems that deliver correct result for any (arbitrary high) probability  $R$

**Report here: cum grano salis!!**

# Two Army Problem (Coordinated Attack)

- p,q processes  
communicate using messages  
messages can get lost  
no upper time for message delivery known  
do not crash, do not cheat
- p,q to agree on action (e.g. attack, retreat, ...)
- how many messages needed ?
- first mentioned: Jim Gray 1978

# Two Army Problem (Coordinated Attack)

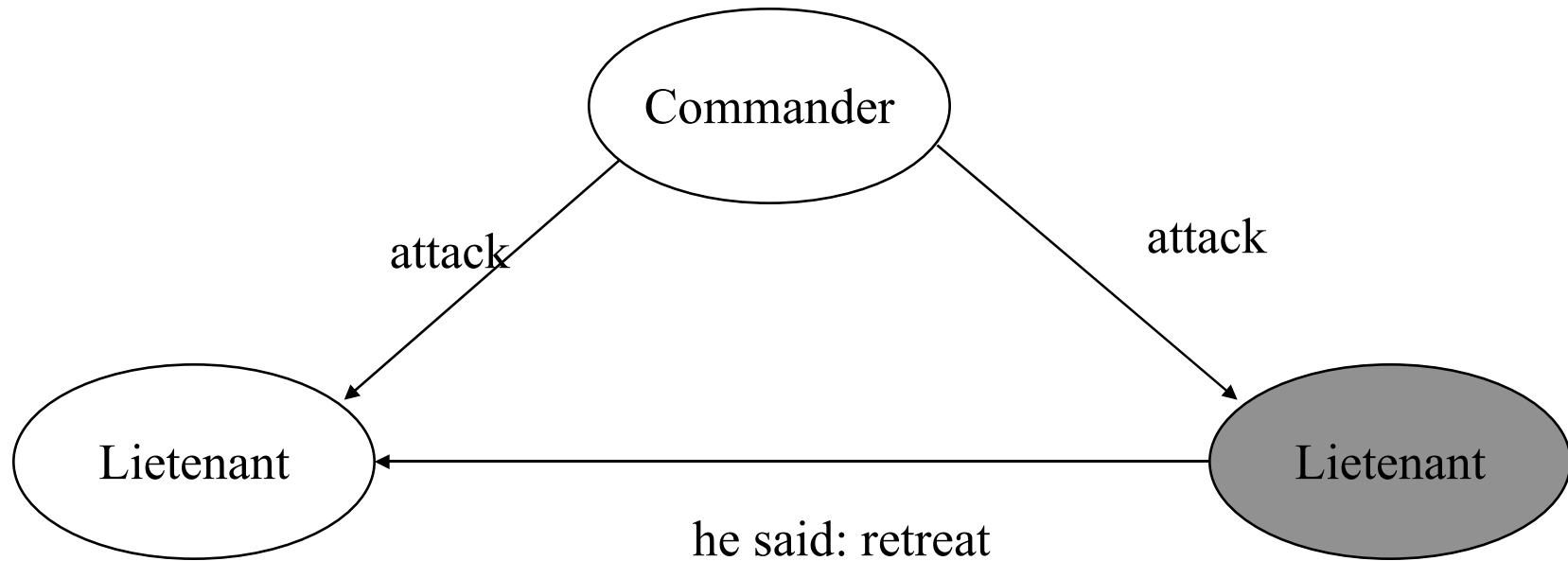
- Result:  
there is no protocol with finite messages
- Prove:  
by contradiction  
assume there are finites protocols  $( m_{p \rightarrow q}, m_{q \rightarrow p} )^*$   
choose the shortest protocol MP,  
last message MX:  $\underline{m}_{p \rightarrow q}$  or  $\underline{m}_{q \rightarrow p}$   
MX can get lost  
 $\Rightarrow$  must not be relied upon  $\Rightarrow$  can be omitted  
 $\Rightarrow$  MP not the shortest protocol.  
 $\Rightarrow$  no finite protocol



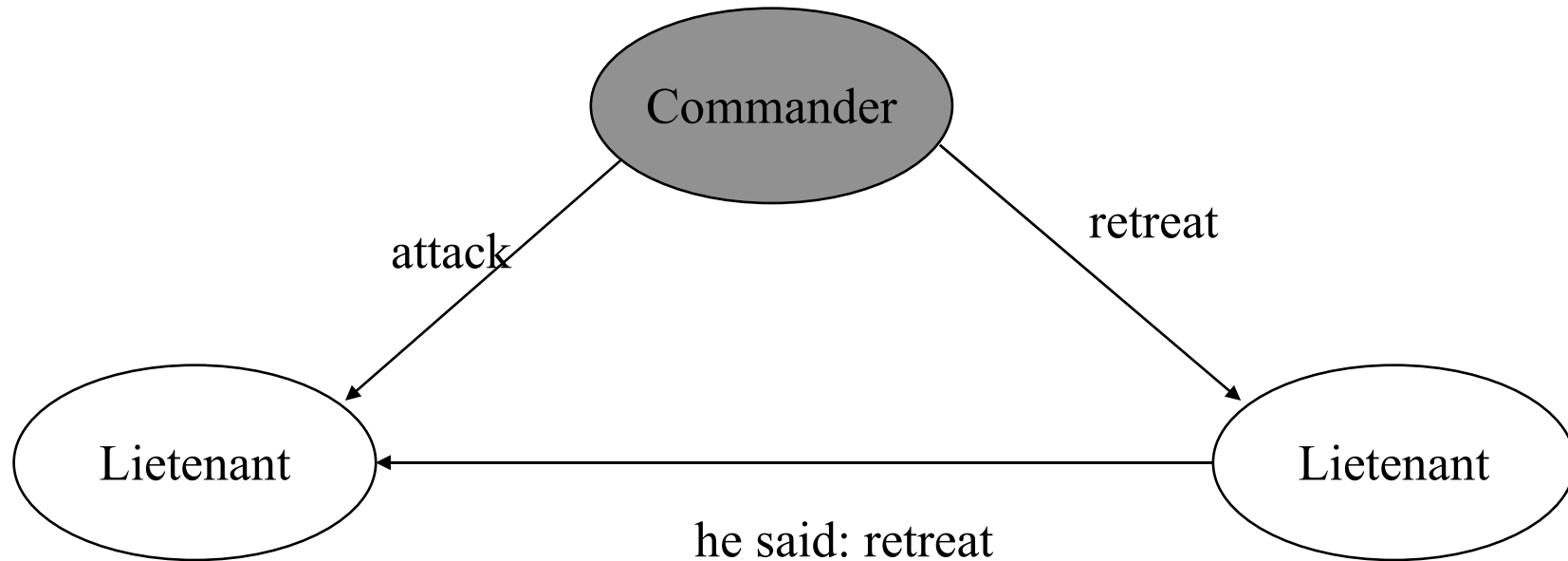
# Byzantine Agreement

- $n$  processes,  $f$  traitors,  $n-f$  loyals  
communicate by reliable and timely messages  
(synchronous messages)  
traitors lie, also cheat on forwarding messages  
try to confuse loyals
- goal:  
loyals try to agree on action (attack, retreat)  
more specific:  
one process is commander  
if commander is loyal and gives an order,  
loyals follow the order  
otherwise loyals agree on arbitrary action

### 3 Processes: 1 traitor, 2 loyal

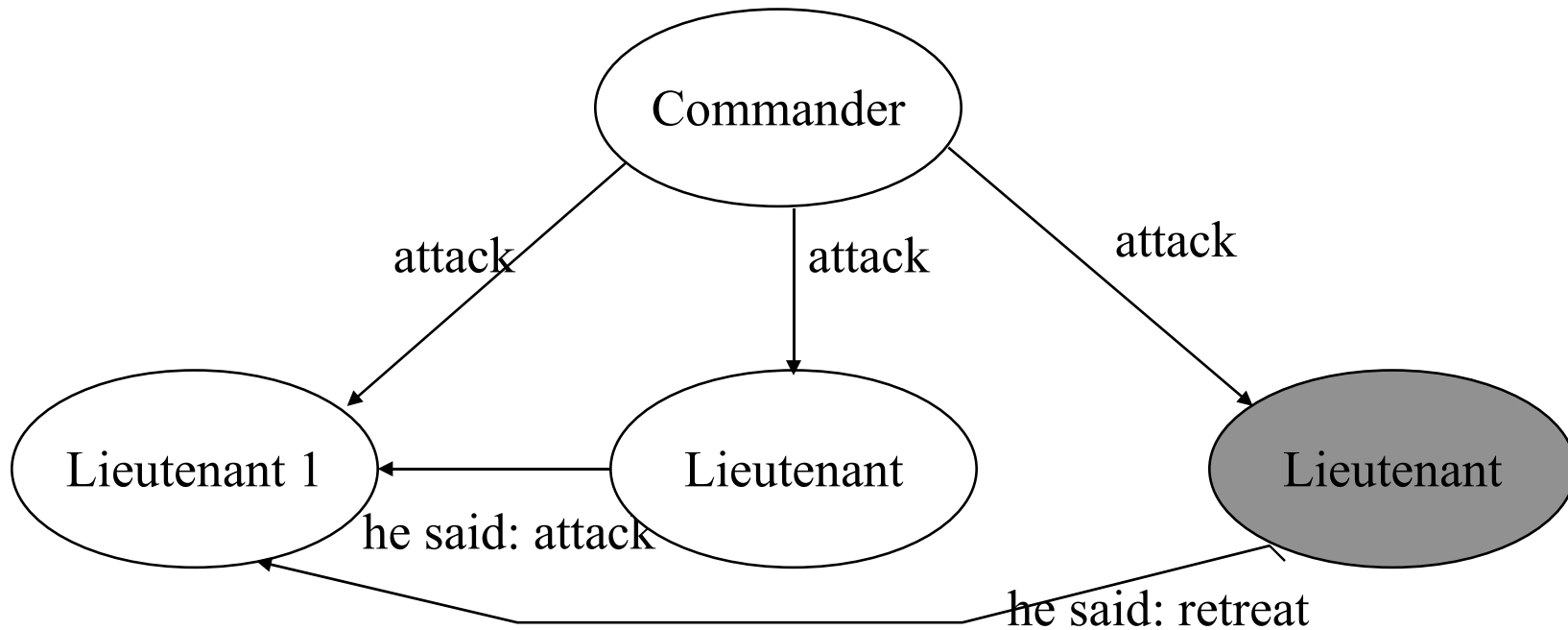


## 3 Processes: 1 traitor, 2 loyal

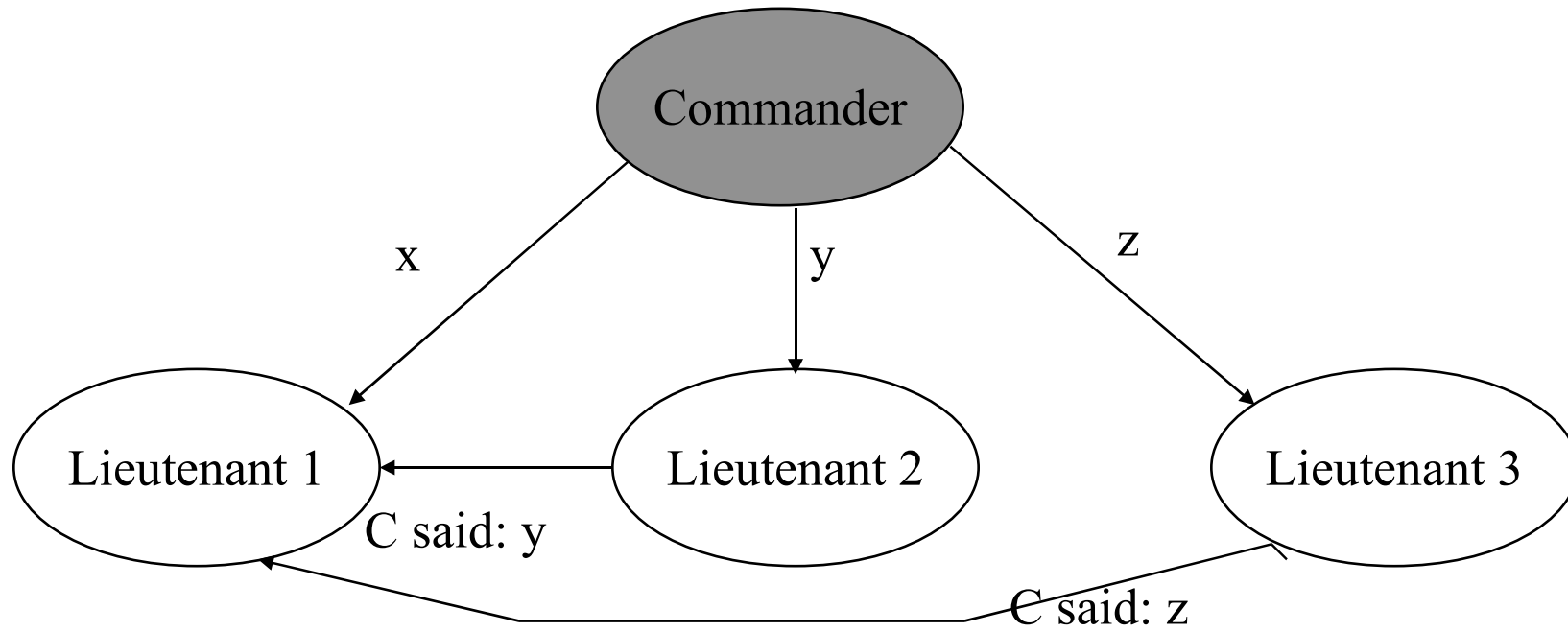


- 3 processes not sufficient to tolerate 1 traitor

# 4 Processes



## 4 Processes



- all lieutenant receive  $x, y, z$
- can decide
- General result:  
 $3f + 1$  processes needed to tolerate  $f$  traitors

## To take away

modeling is very powerful

extreme care needed to do it correctly